Catapult Glider Launching Theories

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Introduction

In order to get catapult glider flight time up, it is clear that getting the glider to high launch heights is very important. The catapult launch for both indoor and outdoor catapult gliders is very similar, but there are a few differences. In indoor FF it is an obvious advantage to get the model to the top of the ceiling of a building. In outdoor FF the ceiling is unlimited but there are some rule restrictions to the amount of rubber that can be used. This paper will try to be applicable to both events since the fundamental laws of physics stay the same. Understanding the parameters/issues that increase launch highs should help get better flight times.

The Perfect Launch

First let us imagine "the prefect launch". Envision it as the glider going straight up vertically until the glider runs out of speed and comes to a stop. Just as it stops the model does a snap and now it is flying level and at its gliding speed. The perfect launch has and can be done but it takes practice and a well-trimmed model.

The Simplified Launch Equations

Now one must start to think about the perfect launch and how it related to simple physics. To start the model has no energy. You stretch the rubber band out and the model goes along. Now all the energy is stored in the rubber band. You release the model and the energy from the band gets transferred into the model, mainly into kinetic energy or motion. Then as the model is climbing it is being resisted by gravity and drag. Some simple physics equations can predict what height the model should obtain if drag is ignored. So if it is assumed all the energy in the rubber band gets converted into kinetic energy and that in turn gets converted into potential energy then the following equations apply:

Potential Energy equation: E=m*g*h; where E=energy, m=mass of model, h=height, g=gravity
Kinetic Energy equation: $E=\frac{1}{2}*m*v^2$; where E=energy, m=mass of model, v=velocity
Work equation: E=F*d; where E=energy, F=Force of the Rubber Band, d=distance.

Using these equations the stored energy in the rubber band can be converted into kinetic energy:

$$E=\frac{1}{2}*m*v^2$$

Solving for v:

$v=(2*E/m)^{.5}$ where E is the energy in the rubber band. - (Equation 1)

This rubber band energy can be converted into height:

$$E=m*g*h$$

Solving for h:

$$h=E/(m*g) \text{ - (Equation 2)}$$

One can obtain a weight vs distance graph by taking a loop of rubber and hanging some weights on it, then measuring the distance the rubber loop stretched. Below is a graph for a ¼" band with a 7" loop.

Image 1

Now some converting needs to be done. Mass in grams is not a unit of force and force is quantity of interest. One must remember that the mass hung on the loop had gravity pulling it down. Therefore to calculate the force one must multiply the mass by the acceleration due to gravity. The earth's gravitation acceleration is 32.2 ft/sec^2 or 386.4 in/sec^2. So by converting one can obtain a force vs rubber band stretch distance graph like the one below:

Image 2

The work or energy equation for the rubber band is the E= F*d. To use this equation is this form the force must be constant throughout the distance and the graph clearly shows this is not true. The force is changing with distance and by using some calculus it is known the total energy in the rubber band is the integral or area under the curve. For a curve which is a straight line which this rubber band data happen to be, the area under the curve is a triangle:

$$E={}^{1}/_{2}*(F_2-F_1)*(d_2-d_1) \text{ - (Equation 3)}$$

For a reasonable band stretch of 48 inches we can plug in the numbers into equation 3 and obtain the energy in the rubber band:

$$E= .5*(1{,}043{,}872-0)*(48-7)=21{,}399{,}376 \text{ g-in}^2/\text{sec}^2.$$

Now knowing the mass of the model one can calculate the velocity from equation 1. For this example it will be assumed the mass of the model is 10 grams. $v=(2*21{,}399{,}376/10)^{.5}=2068.8$ in/sec

Converting to miles per hour: v=2068.8*.0568=117.5 mi/hr

Now using the potential energy equation (equation 2) one can predict the height of the model. Plugging in the numbers simple physics predicts: h= 21,399,376/(386.4*10); where h is the height in in, E is the energy in gram-inch^2/sec^2, m I the mass of the model in grams.

Or: h=5538.1in or 5538.1/12=461.5 ft

This is a very conservative number since it assumes no drag and all the energy from the band is converted to height. But it does give an upper limit to the height one could expect.

Detailed Approach including drag

A more detailed analysis was desired in order to better predict catapult glider launch heights. It was clear that drag is an important factor in determining the glider launch height. So a more complicated method basically known as numerical integration was chosen to perform these calculations. This method breaks the problem into a bunch of steps or segments, and performs detailed calculations on each step. It was choose to split the problem into a number of time steps. Distance or velocity steps could have been used as well, but time steps are the simplest for this type of problem. If comparatively large time steps are used, then big errors may develop. But, if smaller time steps are used, the errors are reduced and the results of the calculation will be more accurate. The disadvantage of using smaller time steps is that computation times are increased. With the speed of current computers, this is hardly noticeable.

The Work Engine

To perform these calculations a work engine or program was needed that could easily handle multiple equations and quickly display results. A spreadsheet program like Microsoft Excel performs this job well. This is a popular program that many people have access to and are familiar with using. Therefore, Excel was chosen as the work engine.

Basic Equations

Here is a summary of the equations used.

Reynolds Number: Re=Chord of the wing*Density of Air*Velocity of the Air/Viscosity of Air. Or: $Re=d*\rho*v/\nu$
Newton's 2nd Law: Force= Mass* Acceleration or F=m*a
Basic law of motion of an object: Distance=.5*Acceleration*time^2+ initial Velocity *time+ initial distance Or: $d = 0.5a*t^2 + v_i*t + d_i$
First Derivative of the Basic law of motion: Velocity = Acceleration*time + initial velocity or $v=a*t + v_i$
Linear Force equation (Hooke's Law): Spring Force = Spring Constant * Distance or F = K*x.
Basic Drag Formula: Drag Force = 0.5*Drag Coefficient*Density*Area*Velocity^2 or $F = 0.5*\rho*v^2*A*Cd$
Cd for smooth flat plates[1]: For turbulent flow Re >= 2000 $Cd=.074*Re^{-.2}$ and for laminar flow $Cd=1.328*Re^{-.5}$

Assumptions

As with any calculations some assumptions are made. If these assumptions are not valid, then errors are introduced into the results. Reducing the number of assumptions in a calculation can make improvement. Below is a list of the assumptions made.

- The glider is assumed to travel in a straight line and at the launch angle inputted.
- The rubber band is assume to release all of the energy that was put into the rubber band during the stretching process and that the rubber band release this energy linearly (according to Hooke's Law).
- The mass of the rubber band is ignored.
- There is no induced drag from lift during the launch. Typically the model is trimmed so during the launch portion of the flight the wings effectively don't develop lift and therefore the induced drag is very close to zero.
- It is assumed that there is no large moment developed by the wing (i.e. Fixed flaps).
- The drag force is assumed to be of two generic types during the launch of a glider. Frontal area drag (Pressure drag) is one which is dependent on the cross sectional area of the glider that is resisting the wind. Frontal drag is also highly dependent on the shape of the object. The other type is skin friction drag, which depends on the surface area of the model and how smooth the surface is. Both forms of drag are calculated using the basic drag formula. It is only the drag coefficients and areas that are different between the two types.

- The skin drag coefficient is assumed to be a smooth flat plate for all surfaces of the glider. The average Reynolds number for the wing is assumed for all surfaces and either laminar or turbulent flow is assumed when calculating the skin drag coefficient.

The Input Parameters
The input parameters needed to perform the calculation on the catapult glider launch height are given below. The first column in this table refers to the cell number in the excel spreadsheet.

Insert Table 1

Insert Table 2

Summary of the Results
A lot of detailed calculations are need to determined two values, which are of most interest. The first is how high the model is launched and the second is the maximum velocity the model attained. The maximum velocity is of interest because this number gives an understanding of how strong the model needs to be built.

Running the Spreadsheet
The input parameters are located in cells A5 thru A17 and are pretty much self-explanatory. Excel is an easy program to run, to change a parameter in the spreadsheet click in the appropriate cell. After the box has been highlighted type in the new desired parameter value. Press the return key to have the spreadsheet recalculate everything with the new parameters. The spreadsheet included in this document has the parameters for a Stan Buddenbohm's, Upsweep design.

The figures below illustrate the discussion laid out on in the paper. If you wish to try the simulation for yourself visit the NFFS web site and down load the program:

http://www.freeflight.org/store/2005symposium.htm

Insert Table 3

It should be noted that if you make the time interval real small the spreadsheet runs out of lines before the velocity of the model becomes zero and the maximum height is reached. To correct this one needs to copy the last line of the spreadsheet then paste this line many times to extend the spreadsheet until the velocity is able to reach zero. Also note too large of a time interval will give you inaccurate results. The time interval of about .001 second appears about right for the typically glider parameters.

Conclusions
The glider parameters inputted in the spreadsheet won the 2001 NATS in Johnson City. The ceiling height at Johnson City, Tn, USA is 116 ft. The glider did hit the rafters of the building a few times, but it probably was not at the highest point. It probably achieved 100-110 ft of height during launch. The drag coefficient was taken from a fluid flow text book. It should also be noted that if you zero out the drag in the spreadsheet, use a launch angle of 90 degrees, a release height of zero, and the same rubber band parameters as in the example for the Simple Launch Equations you get the following summary. Maximum velocity is 119 mi/hr and maximum height 478 ft. These numbers are not very different (about 4%

difference) from the Simple Launch equations numbers. This along with some experimental data gives some confidence that the detail approach is calculating reasonable numbers. This spreadsheet can be a useful tool to understand how to improve catapult glider designs. Varying the input parameters and observing their effect on the output can make intelligent design decisions.

References

1. Fluid Mechanics, 6^{th} Edition. Victor L. Streeter, E. Benjamin Wyle. Pg. 275, Fig. 5.17